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Review of Mateusz Pyzik's Doctoral Dissertation

To whom it may concern,

Mateusz Pyzik's doctoral dissertation titled

Reflection of Continuation-Passing Style in Calculi of Delimited Control

presents contributions in the study of control operators, continuation-passing style translations, as well as direct-style translations as their inverse. The doctoral thesis comprises of three publications at respected venues. The first two publications are co-authored with the supervisor Dariusz Biernacki and Filip Sieczkowski. The last publication lists Mateusz Pyzik as sole author.

Summary of the Dissertation

The CPS translation of delimited control operators gives rise to continuations that are composable, rather than abortive. The aim of the thesis is to find an inverse of the CPS translation which, together with the CPS translation, forms a reflection. Doing so enables compiler writers of languages with delimited control operators to implement optimizations in direct style, rather than going to CPS and optimizing there. The thesis presents the first reduction-preserving direct-style translation for delimited continuations. It goes on to introduce a direct-style translation for the control operator *shift0*. Finally, the image of the CPS translation of *shift0* is closed under reduction, yielding a full subcalculus of λ. The resulting calculus has many desirable theoretical properties, such as confluence and the translations forming reflection.

Chapter: Introduction

Chapter 1 presents the rationale behind delimited control operators in general and shift0 in particular. It further introduces continuation-composing style (CCS) as the image of the CPS translation of delimited control operators. The introduction draws its motivation from Sabry and Wadler (1997), who argue that reflections offer a valuable mechanism for compiler authors, enabling loss-free conversion between source (kernel) and target, and back. While direct-style translations for undelimited control operators exist, they do only form Galois connections, not reflections. Also, no direct-style translations (with the notable exception of Kameyama and Hasegawa (2003), who developed an equational theory for shift but do not consider a reduction theory) have been studied for delimited control operators; even less so for *shift0*, which gives rise to "stacked continuations". The thesis sets out to study direct-style translations for delimited control operators that form reflections with their continuation-passing counterparts.

The first chapter goes on and introduces relevant technical terminology such as control effects, continuations, and the difference between abortive and composable continuations. It continues to introduce mathematical concepts of equational correspondences, Galois connections, and reflections. Importantly, reflections factor into an inclusion and an order isomorphism, giving rise to a kernel in the source language that admits the same optimizations as the image of the translation.

Finally, the main contributions of the remaining thesis are summarized culminating in a reflection and directed axiomatization for the delimited control operator shift0.

Chapter 2: A Reflection on Continuation-Composing Style
Chapter 2 studies continuation-composing style as the image of the CPS translation for the delimited control operator *shift*. It presents a direct-style (DS) translation as the right inverse for the CPS translation. The two translations form a reflection.

The chapter starts by introducing λ_{cS}, a variant of Moggis computational lambda calculus extended with *shift*. Then, a CPS translation is given from λ_{cS} to λ_{cS}^{*}. The CPS translation follows the standard techniques to avoid administrative redexes. The image of the translation λ_{cS}^{*} is given in the style of literal movement grammars to identify usages of continuation variables. Dually, a DS translation from λ_{cS}^{*} to a subset of λ_{cS} (the "kernel") is given. Following Sabry and Wadler, the kernel is identified by normalizing with respect to associativity of let bindings. Reduction of the kernel is adjusted to maintain this normal form.

It is shown that the DS translation is a right inverse to the CPS translation and that the reflection can be factored into an inclusion and an order isomorphism. It corresponds to a normalization procedure that let-binds non-trivial subterms similar to fine-grain call-by-value.

The chapter then goes on and studies λ_{S}, a more traditional subcalculus of λ_{cS}, without let-bindings. It is shown that λ_{S} is isomorphic to λ_{S}^{*}, which in turn is shown to be a subcalculus of λ_{cS}^{*}.

Chapter 3: Reflecting Stacked Continuations
While the second chapter studied reflection into a calculus with the control operator *shift*, the third chapter focuses on *shift0* and presents CPS and DS transformation that form a reflection.

It proceeds by presenting $\lambda_{c\$}$, a calculus featuring the control operators *shift0* and *dollar*, equipped with a novel reduction theory, which expresses the capture of continuations as local interactions. Notably, the naming rule of Sabry & Wadler is generalized to nameless contexts, which admits an elegant presentation. The reduction theory of the calculus is based on a new rule to reassociate let-bindings and the *dollar* operator.

The new fine-grained reduction semantics of $\lambda_{c\$}$ is non-standard. Section 4 of Chapter 3 in contrast presents a calculus $\lambda_{\$}$, with a standard semantics and CPS translation for shift0. Embeddings and inverses are presented, and it is shown that $\lambda_{c\$}$ simulates $\lambda_{\$}$. Interestingly, all but one axiom ($\eta_{\$}$) of Materzok can be derived from the rules. The image of the CPS translation is precisely captured in a CPS calculus. In contrast to the calculus from Chapter 2 for shift, there is no need to distinguish functions and continuations. Like in Chapter 2, a DS translation is introduced, and it is shown that it is a right-inverse to the CPS translation. Using the DS translation, the direct-style kernel of $\lambda_{c\$}$ can be identified.

Finally, various meta-theoretic properties are established; first and foremost, that CPS and DS translation form a reflection.

Chapter 4: Call-by-name is Just Call-by-value with Delimited Control
The last chapter establishes the relation between a calculus with the delimited control operator *shift0* and the call-by-name lambda calculus. In particular, the goal of the chapter is to build on the previous chapter and present a complete axiomatization (whereas Chapter 3 was missing axiom $\eta_{\$}$). The newly introduced calculus $\Lambda_{\$}$ features a unary version of the *dollar* operator. While the reduction $\$(S_0(V)) \to V$ appears as standard, $S_0(\$(M)) \to M$ is surprising. A CPS translation to a (call-by-name) lambda calculus with beta and eta reductions is presented.

As a next step, an equational correspondence of the new calculus $\Lambda_{\$}$ with $\lambda_{\$}$ is established to show that the non-standard unary control operator is well-behaved. It is argued that establishing a Galois connection based on reductions is not possible. The most interesting rule in Figure 5 is the inverse translation $\pi(S_0(M))$, which fixes the evaluation order.

Finally, a reflection between $\Lambda_{\$}$ with λ is presented. Surprisingly, the range of the CPS translation closed under reduction is the entire set of lambda terms. Furthermore, every lambda term can be reflected back into $\Lambda_{\$}$. Various meta theoretic properties are established, culminating in the statement that the direct-style translation and the CPS translation form a reflection.

The kernel within $\Lambda_{\$}$ contains the whole lambda calculus, albeit in a somewhat non-standard representation.

Assessment
The first, introductory, chapter aims to provide a high-level overview over the developments and contributes some context that helps appreciate the individual contributions that follow. It also introduces important technical concepts. It is a bit too short to be truly self-contained. The first chapter presents itself in parts in an informal language, which in a few places impedes reception of the technical contents. The description of future work in Section 1.7 is surprisingly short, given the significant contributions in this thesis.

Chapters 2 improves significantly over the state-of-the-art by presenting the first reduction-preserving reflection of a delimited control operator. One important result of the technical treatment is a one-pass direct-style translation for λ_S.

The contributions of Chapter 3 are not less significant as it presents a reflection for *shift0*, which has a close connection to algebraic effect handlers. The recent practical and theoretical interest in effect handlers renders the contributions of the third chapter even more important.

The fourth chapter represents a theoretically interesting conclusion to the study of reflection of delimited control. The calculus allows for a complete axiomatization and the full (by-name) lambda calculus can be reflected into a kernel calculus.

In summary, the doctoral thesis presents a sound and comprehensive study of reflections for delimited control operators, starting from *shift*, going over *shift0*, to a full axiomatization. In his thesis, which presents multiple strong metatheoretical results, Mateusz Pyzik demonstrates his ability to study interesting properties of novel calculi. The textual quality of the two peer reviewed core chapters (2 and 3) is notably better than that of the enclosing Chapters 1 and 4. The mathematical rigor, however, remains very high throughout the thesis.

It is my great pleasure to recommend the acceptance of Mateusz Pyzik’s thesis. His contributions significantly improved the state-of-the art and I am excited to read future publications by Mateusz.

Sincerely,

Jun.-Prof. Dr. Jonathan Immanuel Brachthäuser